SAINTS COLLEGE LIBRARY
TRIVANDRUM

(Pages : 4)

H – 6260

Reg. No. .....

Name : .....

Fifth Semester B.Sc. Degree Examination, December 2019

First Degree Programme Under CBCSS

Mathematics

Core Course — V

MM 1542 : COMPLEX ANALYSIS — I

(2014 admn. onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Express $\frac{1}{i}$ in the form $a+bi$.

2. What is the geometrical representation of $\{z : z = \bar{z}\}$?

3. Define Cauchy sequence.

4. Define region.

5. Define analyticity at a point.

6. If $f(z) = u + 7i$ is an analytic function, then u is ...

7. State Uniqueness theorem for power series.

8. Give an example of an entire function.

P.T

9. Find $\int_C z^2 dz$, where C is any smooth closed curve.

10. State Integral theorem.

SECTION – II

Answer any **eight** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Find square roots of $2i$.

12. Show that $\sum_{k=1}^{\infty} \frac{1}{k+i}$ diverges.

13. Prove that $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$.

14. Prove that $\{z_n\}$ converges if and only if $\{z_n\}$ is a Cauchy sequence.

15. Prove that power series are infinitely differentiable within their domain of convergence.

16. State sufficient conditions for differentiability of $f(z)$ at a point z.

17. Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n z^n}{n!}$.

18. Verify that $f(z) = z^2$ is analytic in the complex plane.

19. Prove that $\sin 2z = 2 \sin z \cos z$.

20. Let $f = u + iv$ is analytic in a region D and u is a constant. Prove that $f(z)$ is a constant.

21. Prove that $e^z \neq 0$, for any $z \in c$.

22. Let C_1 and C_2 are smoothly equivalent. Prove that $\int_{C_1} f = \int_{C_2} f$.

SECTION – III

Answer any **six** questions from among the questions 23 to 31. These questions carry **4** marks each.

23. Prove that a polynomial $P(x, y)$ is analytic if and only if $P_y = iP_x$.

24. Show that $f(z) = |z|^2$ is differentiable at $z = 0$, but nowhere analytic.

25. Let $f(z) = \begin{cases} \frac{z^2}{z}, & when\ n \neq 0 \\ 0, & when\ z = 0 \end{cases}$. Even though partial derivatives of the component functions exists and satisfy Cauchy-Riemann equations at $z = 0$, show that $f(z)$ is not differentiable at $z = 0$.

26. Show that the power series $f(z) = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \ldots$. is equal to $\sin z$.

27. Evaluate $\int_C f(z)dz$, where $f(z) = \frac{1}{z}$ and C is $z(t) = R\cos t + iR\sin t,\ 0 \leq t \leq 2\pi$, $R \neq 0$.

28. Let C be a smooth curve; let f and g be continuous functions on C; and let α be any complex number. Prove that $\int_C [f(z) + g(z)]dz = \int_C f(z)dz + \int_C g(z)dz$.

29. Show that, if f is a continuous real-valued function and $f << 1$, then $\int_{|z|=1} f << 4$.

30. Suppose f is the derivative of an analytic function F-that is, $f(z) = F'(z)$, where F is analytic on the smooth curve $C : z(t), a \leq t \leq b$. Prove that $\int_C f(z)dz = F(z(b)) - F(z(a))$.

31. State and prove closed curve theorem.

SECTION – IV

Answer any **two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. (a) Suppose $f(z) = \sum_{n=0}^{\infty} C_n Z^n$ converges for $|z| < R$. Prove that $f'(z)$ exists and equals to $\sum_{n=0}^{\infty} nC_n Z^{n-1}$ throughout $|z| < R$.

(b) Prove that a region S is polygonally connected.

33. (a) Show that the function $f(z) = xy + iy$ is nowhere analytic.

(b) Suppose f is analytic in a region and $f' \equiv 0$ there. Show that f is constant.

(c) If real part of an analytic function is a constant, then show that the function is a constant.

34. (a) State and prove necessary conditions for differentiability of $f(z)$ at point z.

(b) Prove that $2\sin z_1 \cos z_2 = \sin(z_1 + z_2) + \sin(z_1 - z_2)$.

35. State and prove Rectangle theorem.
